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Application of Analytical Methods About Equations of Stokes for Transient Condition in Flow Over Oscillating Plane and Oscillating Flow Over Stationary Plane

Edris Ghonoodi*, Davood Domeiri Ganji

Departement of Mechanical Engineering, Babol University of Technology, Babol, Iran

Email address:
e.ghonoodi@gmail.com (E. Ghonoodi), ddg_davood@yahoo.com (D. D. Ganji)

*Corresponding author

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Abstract: In this study, two highly accurate and simple analytical methods (known as semi exact solutions), the variational iteration method (VIM) and Adomian's decomposition method (ADM) are applied for illustrating transient condition of viscous fluid flow over oscillating plane and also oscillating viscous fluid flow over stationary plane. The flow of an incompressible viscous fluid, caused by the oscillation of a flat wall and also the flow of an oscillating fluid flow over stationary wall are considered by Navier-Stokes equations and are subjected to the behavior of fluid flow in boundary layer at transient condition. The main purpose of this article is to solve transient Navier-Stokes first and second equations in new mathematical solving method which is called semi exact solutions where in each case, the velocity of viscous fluid is determined as a function of time and also vertical distance from plane in boundary layer at transient condition. Results reveal the boundary layer thickness and also the transient fluid flow velocity in boundary layer and even more it shows that the (VIM) and (ADM) methods are very effective and accurate in comparison with the exact solution results. The results demonstrate the velocity of fluid in boundary layer as a function of displacement and time and it is shown that in different time, the value of velocity obtained by "VIM" and "ADM" solving methods is almost equal to velocity which is derived from exact or numerical solutions. So the main background and reason of applying the mentioned methods is to verify the accuracy of "VIM" and "ADM" in solving different fluid mechanics equations especially Navier-Stokes equations.

Keywords: Analytical Methods, VIM, ADM, Viscous Flow, Oscillating Wall, Stationary Wall, Transient Condition, Stokes Equation

1. Introduction

The investigation of considering viscous fluid flow over oscillating plane and oscillating viscous fluid flow over stationary plane come from the Stokes equations during the course of his study of pendulum friction.

the Stokes boundary layer, or oscillatory boundary layer, refers to the boundary layer close to a solid wall in oscillatory flow of a viscous fluid or, it refers to the similar case of an oscillating plane in a viscous fluid at rest, with the oscillation directions parallel to the plane.

By solving of mentioned Stokes equation, the transient velocity of fluid is derived according to its height from solid wall in boundary layer.

The classical analytical solution of this problem is given by Panton [1] which two types of answers for Stokes equation consist of steady state and transient conditions, Panton has also presented a closed-form solution to the transient component of Stoke's problem using the steady state component as the initial profile [9].

Erdogan obtained an analytical solution describing the flow at small and large times after the start of the boundary by the Laplace transform method [2]. He has considered the flow of a viscous fluid produced by a plane boundary moving in its own plane with a sinusoidal variation of velocity. He obtained the steady solution and also the transient solution by subtracting the steady state solution from the starting solution.

In this study it is tried to solve the Navier-Stokes equations by semi exact solution methods of VIM and ADM.

The concept of variational iteration method (VIM) is proposed by He and is a modified general Lagrange's multiplier method [3]. VIM has been favorably applied to various kinds of nonlinear problems. The main feature of this method is in it flexibility and ability to solve nonlinear equations. Accurately and conveniently with linearization assumption is used as initial approximation or trial function, and then a more highly precise approximation at some special point can be obtained. This approximation, coverages rapidly to accurate solution [4].

The Adomian decomposition method (ADM) is a non-numerical method for solving nonlinear differential equations, both ordinary and partial [5]. The general direction of this method is toward a unified theory for partial differential equations (PDE). The method was developed by Adomian [5] and has been modified by Wazwaz [6] and recently by Luo [7].

Table 1. *Nomenclature.*

Nomenclature			
u	velocity	Ω	frequency
u_w	wall periodic velocity	y	height from wall
u_0	wall velocity	Y	$Y=\frac{y}{(\frac{\upsilon}{\Omega})^{0.5}}$
U	ratio of u/ u_f	T	time
u_f	fluid velocity	δ	boundary layer thickness
υ	fluid	t	t=ω. T

2. Problem Description and Mathematical Formulations

Consider an oscillating infinite flat plate which has periodic motion along its length as shown in figure 1.

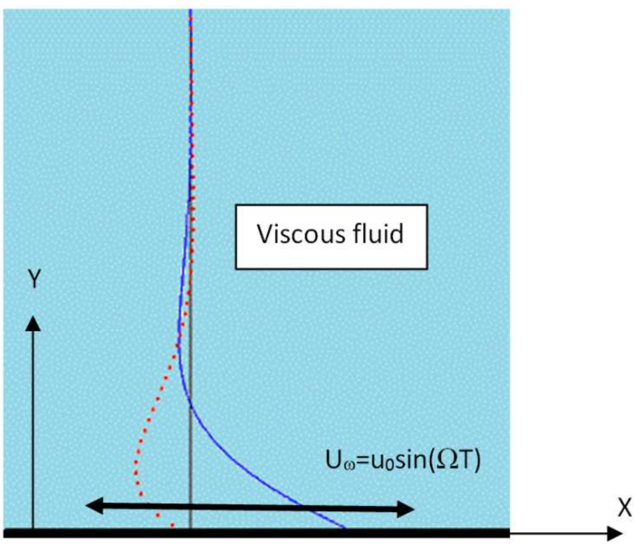


Figure 1. *Viscous fluid flow over oscillating plane.*

The plane velocity is considered as a sine function of time which means that plane starts to oscillate from silence at t=0. The viscous fluid above an infinite flat plane is set in motion by a sudden acceleration of the plate to steady oscillation [8]. The velocity of viscous fluid at (x) direction changes through (y) direction in boundary layer till the fluid height passes the boundary layer thickness (δ). Above the boundary layer, fluid is not being affected by oscillating plane so the fluid velocity will take silence condition.

So by mentioned situation the boundary conditions will be:

$$u(y=0,T)=u_w=u_0\sin(\Omega T) \tag{1}$$

$$u(y=\infty,T)=0 \tag{2}$$

And also to consider transient condition

$$u(y,T=0)=0 \tag{3}$$

In these conditions the initial transition boundary condition equation will be:

$$U(y,T=0)=e^{-y}.\sin(y) \tag{4}$$

Now consider the Navier–Stokes equation in the x direction:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{dp}{dx}+v(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}) \tag{5}$$

By evaluating the problem in X direction only the following terms remain from Navier–Stokes equation and can be written as

$$\frac{\partial u}{\partial t}=v\frac{\partial^2 u}{\partial y^2} \tag{6}$$

It should be mentioned that pressure gradient is considered zero in (x) direction

Now we apply following substitution in Eq. (6) as

$$Y=\frac{y}{(\frac{\upsilon}{\Omega})^{0.5}} \text{ And } U=\frac{u}{u_0} \tag{7}$$

So the Eq. (6) summarized to following as Panton [1]

$$\frac{\partial U}{\partial t}=\frac{\partial^2 U}{\partial Y^2} \tag{8}$$

3. Solving Fluid Equations on Oscillating Flat Plate

3.1. Solving Fluid Equations on Oscillating Flat Plate by Variable Iteration Method

The Variational iteration method (VIM) is proposed by the Chinese mathematician Ji-Huan He [5] and is a modified general Lagrange's multiplier method.

The main property of the method is in its flexibility and ability to solve nonlinear equations accurately and conveniently with linearization assumption is used as initial approximation or trial function, and then a more highly precise approximation at some special point can be obtained. This approximation converges rapidly to accurate solution. The confluence of modern mathematics and symbol computation has posed a challenge to developing technologies capable of handling strongly nonlinear equations which cannot be successfully dealt with by classical methods. Variational iteration method is uniquely qualified to address this challenge. The flexibility and adaption provided by the method have made the method a strong candidate for approximate analytical solution a new iteration formulation is suggested to overcome the shortcoming. A very useful formulation for determining approximately the period of a nonlinear oscillator is suggested.

For better clarify of this method consider the following general nonlinear oscillator differential equations [11]:

$$u' + f(u,u',u'') = 0$$

where u is the velocity and the prime denotes differentiation with respect to t where (t) is time.

According to the variational iteration method (VIM), we can construct a correction functional as follow

$$u_{n+1}(y,t) = u_n(y,t) + \int_0^t \lambda\{u_n'(y,t) - f(u,u',u'')\}d\tau$$

where (λ) is a general Lagrange multiplier, which can be identified optimally via the variational theory, the subscript (n) denotes the th-order approximation.

Now we apply the (VIM) method to the Eq. (6). By this method the linear section of VIM method will be:

$$\frac{\partial U}{\partial t} \tag{9}$$

And $f(u,u',u'')$ part of equation in VIM method is considered as

$$\frac{\partial^2 U}{\partial Y^2} \tag{10}$$

The general Lagrange multiplier is calculated as

$$\lambda = -1 \tag{11}$$

And the initial condition will be

$$U_0(Y,t=0) = e^{-Y}.\sin(Y) \tag{12}$$

The general correction functional of variational iteration method (VIM) for the Eq. (8) is constructed as:

$$u_{n+1}(y,t) = u_n(y,t) + \int_0^t \lambda.\{\frac{\partial U_n\ (y,t)}{\partial t} - \frac{\partial^2 U_n(y,t)}{\partial Y^2}\}d\tau \tag{13}$$

By applying (VIM) method the $U_n(Y,t)$ after multi step corrections yields to:

$$U_0(Y,t) = e^{-y}.\sin(Y) \tag{14}$$

$$U_1(Y,t) = e^{-y}\sin(Y) - 2e^{-y}\cos(Y)t \tag{15}$$

$$U_2(Y,t) = e^{-Y}\sin(Y) - 2e^{-Y}\cos(Y)t - 2e^{-Y}\sin(Y)t^2 \tag{6}$$

$$U_3(Y,t) = e^{-Y}\sin(Y) - 2e^{-Y}\cos(Y)t - 2e^{-Y}\sin(Y)t^2 + \frac{4}{3}e^{-Y}\cos(Y)t^3 \tag{17}$$

$$U_4(Y,t) = e^{-y}\sin(Y) - 2e^{-Y}\cos(Y)t - 2e^{-Y}\sin(Y)t^2 + \frac{4}{3}e^{-Y}\cos(Y)t^3 + \frac{2}{3}e^{-Y}\sin(Y)t^4 \tag{18}$$

$$U_5(Y,t) = e^{-Y}\sin(Y) - 2e^{-Y}\cos(Y)t - 2e^{-Y}\sin(Y)t^2 + \frac{4}{3}e^{-Y}\cos(Y)t^3 + \frac{2}{3}e^{-Y}\sin(Y)t^4 - \frac{4}{15}e^{-Y}\cos(Y)t^5 \tag{19}$$

3.2. Solving Fluid Equations on Oscillating Flat Plate by Adomian Decomposition Method

The Adomian decomposition method (ADM) is a non-numerical method for solving nonlinear differential equations, both ordinary and partial. The general direction of this work is towards a unified theory for partial differential equations (PDE). The method was developed by George Adomian [5]. This method is a semi-analytical method.

This method is useful for obtaining closed form for numerical approximation for a wide class of stochastic and deterministic problems in science and engineering.

For better clarify of this method let's consider a general nonlinear equation in the following form:

$$Lu + Ru + Nu = g \tag{20}$$

where (L) is the highest order derivative which is assumed to be easily invertible, (R) is the linear differential operator of less order than (L), (Nu) presents the nonlinear term and (g) is the source term. Applying the inverse operator (L^{-1}) to the both sides of Eq.(20) we obtain:

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \tag{21}$$

where the function f(x) represents the terms arising from integration the source term g(x), using given condition, for nonlinear differential equations, the nonlinear operator $Nu = F(u)$ is represented by an infinite series of the so-called Adomian polynomials

$$F(u)=\sum_{m=0}^{\infty}A_m \tag{22}$$

The polynomials A_m are generated for all kind of nonlinearity so that A_0 depends only on (u_0), A_1 depends only on (u_0) and (u_1) and so on. The Adomian polynomials introduced above show that the sum of subscripts of the components of (u) for each terms of (A_m) is equal to (n).

The Adomian method defines the solution u(x) by the series

$$u=\sum_{m=0}^{\infty}u_m \tag{23}$$

In the case of "F(u)", the infinite series is a Taylor expansion about "u_0", as follows [15]:

$$F(u)=F(u_0)+F'(u_0)(u-u_0)+F''(u_0)\frac{(u-u_0)}{2!}+F'''(u_0)\frac{(u-u_0)^2}{3!}+\ldots \tag{24}$$

By rewriting Eq. (23) as $u-u_0=u_1+u_2+u_3+\ldots$, substituting it into Eq.(24), and then equating two expressions for F(u) found in Eq.(24) and Eq.(22), formulas for the Adomian polynomials is defined in the form of:

$$F(u)=A_0+A_1+\ldots=F(u_0)+F'(u_0)(u_1+u_2+\ldots)+F''(u_0)\frac{(u_1+u_2+\ldots)}{2!}+\ldots \tag{25}$$

By equating terms in Eq. 25, the first few Adomian's polynomials A_0,A_1,A_2,A_3 and A_4 are given:

$$A_0=F(u_0) \tag{26}$$

$$A_0=F(u_0) \tag{27}$$

$$A_2=u_2F'(u_0)+\frac{1}{2!}u_1{}^2F''(u_0) \tag{28}$$

$$A_3=u_3F'(u_0)+u_1u_2F''(u_0)+\frac{1}{3!}u_1{}^3F'''(u_0) \tag{29}$$

$$A_4=u_4F'(u_0)+(\frac{1}{2!}u_2{}^2+u_1u_3)F''(u_0)+\frac{1}{2!}u_1{}^2u_2F'''(u_0)+\frac{1}{4!}u_1{}^4F^{(4)}(u_0) \tag{30}$$

Now while the (A_m) are known, Eq. (22) can be substitutes in Eq. (21) to specify the term in the expansion for the solution of Eq. (23)

Now we apply ADM method to the Eq. (8), so the parameters will be:

$$u_0(Y,t)=e^{-Y}.\sin(Y) \tag{31}$$

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty}\lambda^i u_n(y,t)\right)_{\lambda=0} \tag{32}$$

While

$$N(u)=\frac{\partial^2U(y,t)}{\partial Y^2} \tag{33}$$

$$u_{n+1}(y,t)=\int_0^t A_n dt \tag{34}$$

After substituting initial condition Eq. (31) in Eq. (32) and Eq. (33) the first polynomial of ADM is obtained by Eq. (34), so for more accurate results the polynomial cycle continues to more steps.

So the Adomian's polynomial functions will be:

$$U_0(Y,t)=e^{-Y}.\sin(Y) \tag{35}$$

$$U_1(Y,t)=-2e^{-Y}.\cos(Y)t \tag{36}$$

$$U_2(Y,t)=-2e^{-Y}.\sin(Y)t^2 \tag{37}$$

$$U_3(Y,t)=\frac{4}{3}e^{-Y}.\cos(Y)t^3 \tag{38}$$

$$U_4(Y,t)=\frac{2}{3}e^{-Y}.\sin(Y)t^4 \tag{39}$$

$$U_5(Y,t)=-\frac{4}{15}e^{-Y}.\cos(Y)t^5 \tag{40}$$

So the results will be

$$U(Y,t)=U_0(Y,t)+U_1(Y,t)+U_2(Y,t)+U_3(Y,t)+U_4(Y,t)+U_5(Y,t)+\ldots$$

$$U(Y,t)=e^{-Y}.\sin(Y)-2e^{-Y}.\cos(Y)t-2e^{-Y}.\sin(Y)t^2+\frac{4}{3}e^{-Y}.\cos(Y)t^3+\frac{2}{3}e^{-Y}.\sin(Y)t^4-\frac{4}{15}e^{-Y}.\cos(Y)t^5+\ldots \quad (41)$$

4. Results and Discussion on Fluid Equations on Oscillating Flat Plate

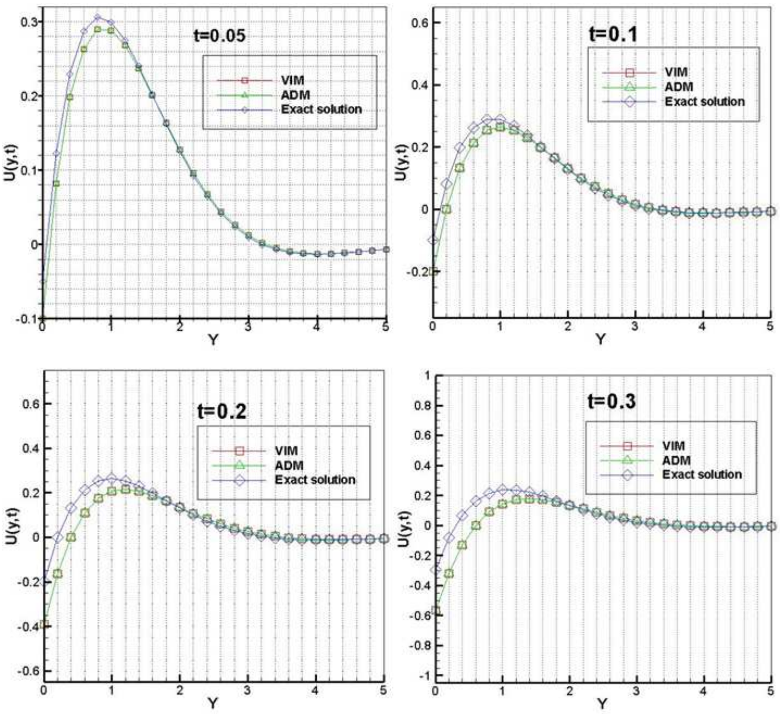
In this study, the behavior of viscous fluid is considered over oscillating flat plane in transient condition by two semi exact solutions of nonlinear equation solving method called (VIM) and (ADM). By applying these two methods to the summarized equation of Navier-Stoks, which is given in Eq. (6), the velocity of viscous fluid is achieved according to fluid height from oscillating plane in boundary layer.

The exact solution of Eq. (6) has been given by Panton [1] as

$$U(Y,t)=e^{-Y}.\sin(Y-t) \quad (42)$$

By considering the variational iteration method (VIM) and Adomian decomposition method (ADM) results, it is observed that the both results are completely same and equal with exact solution of equation in a wide range.

To validate the (VIM) and (ADM) methods, the results are being compared with exact solution in different (t) values.



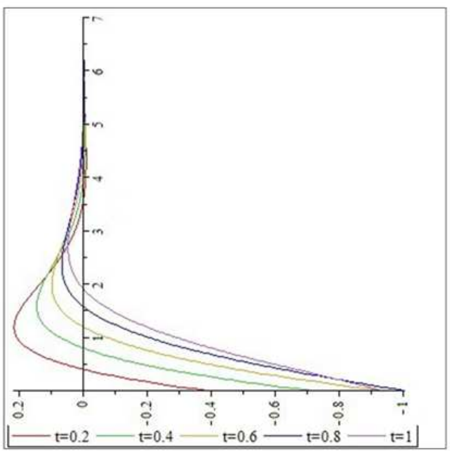


Figure 2. *Boundary layer thickness "δ".*

5. Oscillating Viscous Fluid Flow over Stationary Plane

Now consider the previous problem which discussed in section (2.1) in new case, where the flat plate remains stationary and the stream fluid flow over plate oscillates [14]. Problem shows in following Figure 3:

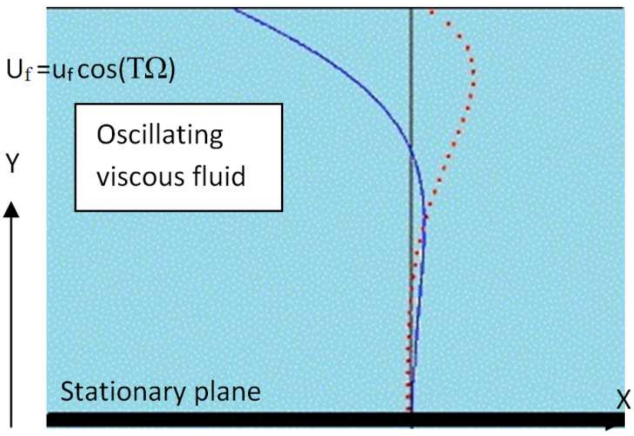


Figure 3. *Oscillating flow over stationary plane.*

At this case the stream viscous fluid flow oscillates above boundary layer at the frequency of (Ω) and the velocity of fluid is affected in boundary layer till the velocity becomes silent at the stationary wall surface [13].

By this case the initial boundary conditions will be:

$$U(Y,t=0)=1-e^{-Y}.\cos(Y) \tag{43}$$

where (t) is considered as:

$$t=T.\Omega \tag{44}$$

and (U) considered as

$$U=\frac{u}{u_0} \tag{45}$$

So by mentioned initial boundary equation

$$U(y=\infty)=1$$

and

$$U(y=0)=0 \tag{46}$$

Now consider the Navier–Stokes equation in the (x) direction like Eq. (5):

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{dp}{dx}+v(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})$$

By evaluating the problem in (x) direction only the following terms remain from Navier–Stokes equation and can be written same as Eq. (6):

$$\frac{\partial u}{\partial t}=v\frac{\partial^2 u}{\partial y^2} \tag{47}$$

It in this case the pressure gradient is also considered zero in (x) direction.

And also by substituting (υ) in (y) as Eq. (7) and considering Eq. (45), the Eq. (47) will be summarized to:

$$\frac{\partial U}{\partial t}=\frac{\partial^2 U}{\partial Y^2} \tag{48}$$

6. Solving Fluid Equations on Oscillating Fluid Flow Above Fixed Plate

6.1. Solving Fluid Equations on Oscillating Fluid Flow Above Fixed Flat Plate by Variable Iteration Method

Now we apply the (VIM) method to the Eq. (48). By this method the linear section of (VIM) method will be:

$$\frac{\partial U}{\partial t} \tag{49}$$

and $f(u,u',u'')$ part of equation in VIM method is considered as

$$\frac{\partial^2 U}{\partial Y^2} \tag{50}$$

The general Lagrange multiplier is calculated as

$$\lambda = -1 \tag{51}$$

And the initial condition will be

$$U_0(Y,t=0)=1-e^{-Y}.\cos(Y) \tag{52}$$

The general correction functional of variational iteration method (VIM) for the Eq.(8) is constructed as:

$$u_{n+1}(y,t)=u_n(y,t)+\int_0^t \lambda.\{\frac{\partial U_n\ (y,t)}{\partial t}-\frac{\partial^2 U_n(y,t)}{\partial Y^2}\}d\tau \tag{53}$$

By applying (VIM) method, the $U_n(Y,t)$ after multi step corrections yields to:

$$U_0(Y,t=0)=1-e^{-Y}\times\cos(Y) \tag{54}$$

$$U_1(Y,t)=1-e^{-Y}\times\cos(Y)-2e^{-Y}.\sin(Y)t \tag{55}$$

$$U_2(Y,t)=1-e^{-Y}\times\cos(Y)-2e^{-Y}.\sin(Y)t+2e^{-Y}.\cos(Y)t^2 \tag{56}$$

$$U_3(Y,t)=1-e^{-Y}\times\cos(Y)-2e^{-Y}.\sin(Y)t+2e^{-Y}.\cos(Y)t^2+\frac{4}{3}e^{-Y}.\sin(Y)t^3 \tag{57}$$

$$U_4(Y,t)=1-e^{-Y}\times\cos(Y)-2e^{-Y}.\sin(Y)t+2e^{-Y}.\cos(Y)t^2+\frac{4}{3}e^{-y}.\sin(Y)t^3-\frac{2}{3}e^{-Y}.\cos(Y)t^4 \tag{58}$$

$$U_5(Y,t)=1-e^{-Y}\times\cos(Y)-2e^{-Y}.\sin(Y)t+2e^{-Y}.\cos(Y)t^2+\frac{4}{3}e^{-Y}.\sin(Y)t^3-\frac{2}{3}e^{-Y}.\cos(Y)t^4-\frac{4}{15}e^{-Y}.\sin(Y)t^5 \tag{59}$$

6.2. Solving Fluid Equations on Oscillating Fluid Flow Above Fixed Flat Plate by Adomian Decomposition Method

Now we apply ADM method to the Eq. (48), so the parameters will be:

$$U_0(Y,t=0)=1-e^{-Y}\times\cos(Y) \tag{60}$$

while

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty}\lambda^i u_n(y,t)\right)_{\lambda=0}$$

$$N(u)=\frac{\partial^2 U(y,t)}{\partial Y^2}$$

So

$$u_{n+1}(y,t)=\int_0^t A_n dt$$

and finally

$$U(y,t)=\sum_{n=0}^{\infty}u_n(y,t)dt$$

and the result will be:

By applying "ADM" method to the Eq. 48, the $u_n(y,t)$ by Adomian's polynomial functions will be:

$$U_0(Y,t)=1-e^{-Y}.\cos(Y) \tag{61}$$

$$U_1(Y,t)=-2e^{-Y}.\sin(Y)t \tag{62}$$

$$U_2(Y,t)=2e^{-Y}.\cos(Y)t^2 \tag{63}$$

$$U_3(Y,t)=\frac{4}{3}e^{-Y}.\sin(Y)t^3 \tag{64}$$

$$U_4(Y,t)=-\frac{2}{3}e^{-Y}.\cos(Y)t^4 \tag{65}$$

$$U_5(Y,t)=-\frac{4}{15}e^{-Y}.\sin(Y)t^5 \tag{66}$$

So the results will be

$$U(Y,t)=U_0(Y,t)+U_1(Y,t)+U_2(Y,t)+U_3(Y,t)+U_4(Y,t)+U_5(Y,t)+...$$

$$U(Y,t)=1-e^{-Y}.\cos(Y)-2e^{-Y}.\sin(Y)t+2e^{-Y}.\cos(Y)t^2+\frac{4}{3}e^{-Y}.\sin(Y)t^3-\frac{2}{3}e^{-Y}.\cos(Y)t^4-\frac{4}{15}e^{-Y}.\sin(Y)t^5... \tag{67}$$

7. Results and Discussion on Fluid Equations on Oscillating Fluid Flow Above Fixed Plate

In earlier study, the behavior of oscillating viscous fluid is considered over stationary flat plane in transient condition by two semi exact solution of nonlinear equation solving method called "VIM" and "ADM". By applying these two methods to the summarized equation of Navier-Stoks which is given in Eq. (48), the velocity of viscous fluid is achieved according to fluid height from stationary plane in boundary layer [10].

The exact solution of Eq. (48) has been given by Fluid Mechanics [3].

$$u(y,t)=u.\cos(\omega t)-u.e^{-ky}\cos(ky-\omega t) \tag{68}$$

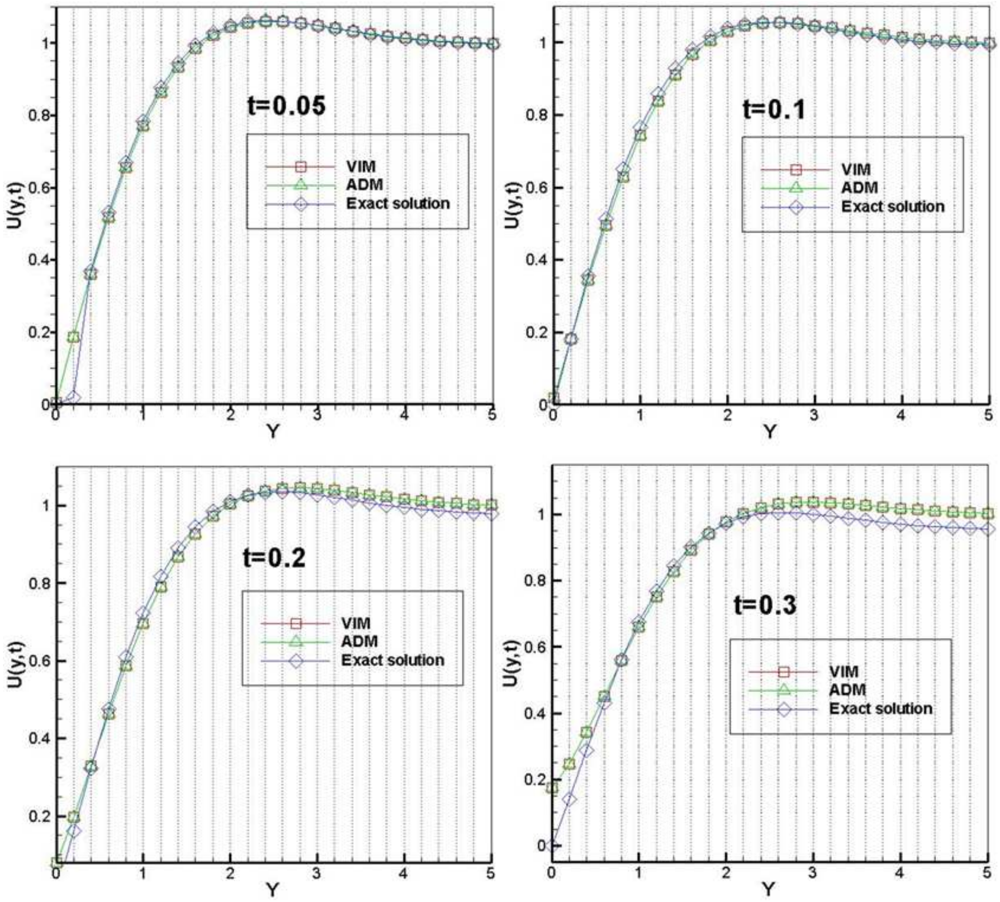
Where

$$k=\sqrt{\frac{\omega}{2\nu}} \tag{69}$$

By considering the variational iteration method (VIM) and Adomian decomposition Method (ADM) results, it is observed that the both results are completely same and equal with exact solution of Stokes equation in a wide range.

To validate the (VIM) and (ADM) methods, the results are being compared with exact solution in different (t) values. It is also concluded that "VIM" and "ADM" are reliable and confidential solutions to solve Navier-Stokes equations and also these methods can be widely used to solve different fluid flow equations.

The main property of these method to compare to other exact solutions, are in their flexibility and ability to solve linear and nonlinear equations accurately and conveniently with linearization assumption in initial approximation or trial function and then a more highly precise approximation at some special point can be achieved [12].



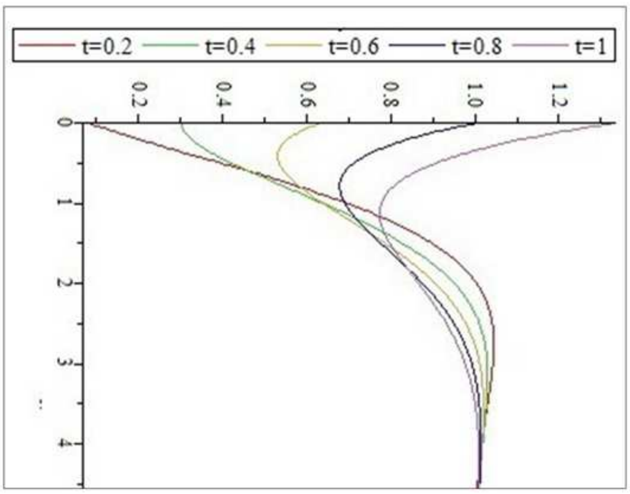


Figure 4. *Boundary layer thickness "δ".*

8. Conclusions

In this paper the behavior of viscous fluid flow was studied in two cases where first the viscous flow was considered above an oscillating plane, and second an oscillating viscous fluid was considered above a stationary plane. In each case the boundary layer thickness was obtained and the velocity of fluid was determined as a function of time and vertical distance from plane in boundary layer in transient condition.

Applying the analytical solution methods as "the variational iteration method (VIM)" and "Adomian's decomposition method (ADM)" to solve the Navier-Stokes equation and comparing with exact solution revealed that these two methods are convenient and accurate so that they can be used in many nonlinear fluid mechanics equations.

Also by obtaining the velocity function of viscous fluid flow above flat plane, the boundary layer thickness “δ” was simply determined in each mentioned cases.

Further study of different cases such as oscillating fluid flow on porous plate or MHD fluid flow on oscillating plate are suggested to be studied for future.

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